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Transport and profile measurements of the diffusive interface in double diffusive convection with similar diffusivities

By T. G. L. SHIRTCLIFFE
Department of Physics, Victoria University of Wellington, New Zealand

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The transport properties of a diffusive interface with diffusivity ratio $\kappa_S/\kappa_T = \frac{1}{3}$ have been measured, using salt and sugar as the diffusing components. The flux ratio is constant and equal to $(\kappa_S/\kappa_T)^{\frac{1}{2}}$. The normalized salt flux is related to the density anomaly ratio $R_\rho = \beta\Delta S/\alpha\Delta T$ by the power law $F_T^* = 2{\cdot}59R_\rho^{-12{\cdot}6}$ over four decades. Optical measurements show that the vertical gradients of concentration of salt and sugar within the interface are those required if molecular diffusion is to account for the whole flux of each component.

1. Introduction

The diffusive interface was first observed by Turner & Stommel (1964) when they showed that water, stabilized by a salt gradient and then heated from below, convects in layers which are separated by thin stable interfaces. This is one example of a type of convection which derives its character from the presence of two components in the fluid, one of which is stabilizing while the other is destabilizing, the components having different molecular diffusivities in the fluid. The most general name which has been given to the phenomenon is double diffusive convection, though for particular choices of the components it has also been called thermosolutal and thermohaline convection. Because of its potential importance in relation to studies of the vertical transport of salt and heat in the ocean, double diffusive convection has received increasing attention recently.

The layers in double diffusive convection are separated by interfaces which can take two forms, depending on whether the destabilizing component has the greater or the lesser diffusivity. The latter case leads to a finger interface, in which salt fingers are observed (Stern 1960, 1969; Stern & Turner 1969; Turner 1967; Shirtcliffe & Turner 1970; Howe & Tait 1970; Linden 1971). The former case is exemplified by the observations of Turner & Stommel (1964) and has been studied, for example, by Turner (1965, 1968), Shirtcliffe (1969*a*) and Turner, Shirtcliffe & Brewer (1970). While a detailed theory of the diffusive interface is still not available, there is substantial evidence in these papers that the molecular diffusivities control the transport through it. None of the previous investigations has suggested that there is a flow of fluid through this type of interface in normal circumstances, although under certain conditions entrainment could become important, and we are left with diffusion as the principal transport mechanism.

Measurements discussed in the latter part of this paper go further, and show that, at least when the interface is not very thin, the mean vertical gradients of both components within the interface are great enough for diffusion to account for the observed fluxes.

Turner (1965) studied the rates of transport of salt and heat in the simple case of one diffusive interface separating two layers. He showed that the transport of heat depends basically on the $\frac{4}{3}$ power of the temperature difference ΔT between the layers, together with a dependence on the density anomaly ratio $R_\rho = \beta\Delta S/\alpha\Delta T$; here $\beta\Delta S$ is the fractional density difference between the layers attributable to the salinity difference ΔS, while $\alpha\Delta T$ is the fractional density difference attributable to ΔT.

On purely dimensional grounds, one would expect also some dependence of the transport on the ratio between the molecular diffusivities involved. In the case of salt and heat, these are in the approximate ratio 1:100, and it is important to make measurements similar to those of Turner with ratios nearer unity, to establish the form of this dependence. The first part of this paper concerns an experiment in which measurements of flux were made under a variety of conditions when the ratio of diffusivities was close to 1:3, using sugar (diffusivity $0{\cdot}52 \times 10^{-5}\,\mathrm{cm^2\,s^{-1}}$ in water at 25 °C) and common salt ($1{\cdot}55 \times 10^{-5}\,\mathrm{cm^2\,s^{-1}}$) as the diffusing components.

2. Flux measurements: methods

The flux measurements were analogous to those which Turner (1965) made on the heat–salt system.

A layer of salt solution was floated on a layer of sugar solution in a tank 60 cm long and 9 cm wide, both layers being about 10 cm deep. It was then necessary to measure, as functions of time, both $\alpha\Delta T$ and $\beta\Delta S$, where $\alpha\Delta T$ is again the difference in mid-layer densities attributable to the faster-diffusing component, in this case the salt, and $\beta\Delta S$ is the corresponding quantity for sugar.

The obvious method of measuring sugar concentration was polarimetry; a simple polaroid analyser sufficed to give first-order values of $\beta\Delta S$ within $\pm 0{\cdot}0004$ when the convection transport was rapid, and within $\pm 0{\cdot}0001$ when it was slow. An optical method was used also to determine $\alpha\Delta T$, since the form of the interface lends itself to this; optical methods also avoid any disturbance to the system such as may be caused, for example, by taking samples for separate analysis. Once $\alpha\Delta T$ was known, a small correction was made to $\beta\Delta S$, which had been calculated initially by assuming zero salt concentration.

In order to find $\alpha\Delta T$, the difference in refractive index Δn between the two layers was measured. Knowledge of the contribution of the sugar then allowed $\alpha\Delta T$ to be found by subtraction. Δn was measured by finding the deflexion of a light beam which passed obliquely through the interface, as shown in figure 1. It is easily shown that, if the lower layer has refractive index n_1 and the upper layer n_2, then

$$\Delta n = \frac{\sin(\theta_1+\theta_2)\sin(\theta_1-\theta_2)}{n_1+n_2}$$

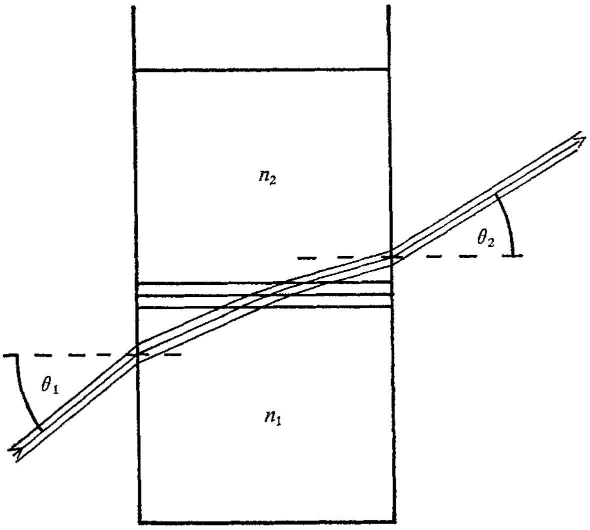

FIGURE 1. The difference in refractive index between the layers was found by measuring the deviation of a light beam passing obliquely through the interface.

provided that variations of refractive index occur only in the vertical direction. This is apparently true for the diffusive interface, with the exception of those narrow regions where plumes of fluid leave the interface. A spectrometer table was mounted with its axis horizontal in order to measure θ_1, which was fixed, and θ_2 as a function of time.

An image of the slit could be seen in the telescope of the spectrometer which was of sufficient clarity to allow θ_1 to be measured to within $\pm 0{\cdot}02^\circ$ when the interface was thin and the convection active, and to within $\pm 0{\cdot}01^\circ$ (the limit of the instrument) when the interface was thicker. As a result, $\alpha\Delta T$ could be determined to within $\pm 0{\cdot}001$ in the former case and $\pm 0{\cdot}0004$ in the latter, where a contribution to this uncertainty comes from the sugar concentration measurement.

The major difficulty of this method of determining $\alpha\Delta T$ was uncertainty as to the precise relationships such as that between the refractive index of a double solution of salt and sugar and the individual concentrations of salt and sugar. The properties of aqueous solutions of either component alone are well tabulated, in the *International Critical Tables*, for example, but no data were available on their combination. A series of 21 measurements with an Abbé refractometer was therefore made with the concentrations of sugar and salt evenly spaced over the field of interest. The solutions were all derived from two standard solutions, one for each component, by mixing and dilution. The measurements were combined into a quadratic expression of the form

$$n = n_0 + a_1 p_T + a_2 p_T^2 + b_1 p_S + b_2 p_S^2 + c p_T p_S, \tag{1}$$

where n_0 is the refractive index of pure water at the same temperature and wavelength, p_T is the mass concentration of salt (i.e. the mass of salt in unit volume of the solution), p_S is the mass concentration of sugar and a_1, a_2, b_1, b_2 and c are constants.

The values determined for the a's and b's in (1) could be checked by setting either p_T or p_S equal to zero and comparing with published tables. It was found that the b's were correct within the experimental limits, but the a's were slightly in error. In any case, the published measurements were more precise, and the a's and b's were, therefore, determined finally by a least-squares fit to values available in the *International Critical Tables*. The usefulness of the present set of measurements was that it established first that a quadratic expression was adequate to cover the field $0 < p_T < 0{\cdot}25\,\mathrm{g\,cm^{-3}}$ and $0 < p_S < 0{\cdot}4\,\mathrm{g\,cm^{-3}}$ to within $\pm 0{\cdot}0002$ in refractive index; and that moreover c is very small, so that the value determined for it should be sufficiently precise. The values finally adopted for the constants in (1) were $a_1 = 0{\cdot}1700\,\mathrm{cm^3\,g^{-1}}$, $a_2 = -0{\cdot}0757\,\mathrm{cm^6\,g^{-2}}$, $b_1 = 0{\cdot}1429\,\mathrm{cm^3\,g^{-1}}$, $b_2 = -0{\cdot}0029\,\mathrm{cm^6\,g^{-2}}$ and $c = -0{\cdot}037\,\mathrm{cm^6\,g^{-2}}$.

A similar uncertainty arose in relation to the density of a double solution as a function of the individual concentrations. Here the difficulty is caused primarily by the nonlinear behaviour of a salt solution. Once again appeal was made to data published in the *International Critical Tables* in order to establish a quadratic relationship similar to (1), that is

$$\rho = \rho_0 + A_1 p_T + A_2 p_T^2 + B_1 p_S + B_2 p_S^2 + C p_T p_S. \tag{2}$$

In this case, however, no comparable experiments were done, and C was arbitrarily set to zero. The other second-degree terms in (1) and (2) are of minor importance compared with the linear terms, and setting $C = 0$ was not expected to introduce a significant error. Nevertheless, the lack of justification for this step was the main uncertainty in the experiment, so for one run a quite different method was used which avoided this step. As will be seen, the results of both methods were entirely consistent, justifying the treatment of C. The values finally adopted for the constants in (2) were $A_1 = 0{\cdot}6932$, $A_2 = -0{\cdot}2066\,\mathrm{cm^3\,g^{-1}}$, $B_1 = 0{\cdot}3837$, $B_2 = -0{\cdot}0111\,\mathrm{cm^3\,g^{-1}}$ and $C = 0$.

The second method of determining $\alpha\Delta T$ and $\beta\Delta S$ was more direct than the optical method. Once again the sugar concentrations were determined by polarimetry, but $\alpha\Delta T$ was found by a direct measurement of the density in each layer and subtraction of the contribution due to $\beta\Delta S$. The densities were determined by weighing a 4 $\mathrm{cm^3}$ sinker suspended in the fluid. A Mettler balance was used, and readings were repeatable to within $\pm 0{\cdot}0005$ g, giving an uncertainty in $\alpha\Delta T$ similar to that achieved with the optical method. This method would have been used from the beginning instead of the optical method, because it is much more straightforward in analysis, were it not for an initial trial which induced some caution. The first problem was the need for some subtlety in designing the sinker. It had to be small compared with the layer of fluid in which it was immersed in order to prevent horizontal density gradients, which could alter the flow pattern, being produced. In addition it had to take the form of a relatively thin vertical cylinder with conical ends, both to prevent the accumulation of fluid of atypical density above or below it, and also to ease the disposal of any bubbles which formed on it.

A further problem with the sinker method derived from the nonlinear behaviour, particularly of a salt solution, which was mentioned above. The relatively large value of A_2 in (2) means that volume is not conserved when a salt solution is mixed with another solution. Thus it was not adequate to measure the density in only one layer and to derive that in the other by comparison with the values at the beginning of the experiment. The mean density in the tank rose progressively as the transport of salt from the upper to the lower layer proceeded, and it was necessary to monitor the density in both layers in order to determine $\alpha\Delta T$. This required a passage of the sinker through the interface from time to time, with a consequent danger of producing an erroneous value for the fluxes.

Ten runs were carried out in all. The optical method was used to measure $\alpha\Delta T$ for the first nine of these, since the risk of systematic errors seemed smaller in this method. An incorrect value of C in (2) would have had most effect when the concentrations of sugar and salt were high, so one run (run 10) was made under such conditions using the sinker method as a check.

3. Flux measurements: results

In this experiment, using salt and sugar as the diffusing components, the total mass of each component was conserved; in this respect it differs from the analogous heat–salt experiment of Turner in which heat was supplied continuously. As a result, the fluxes (in buoyancy units) of salt and sugar through the interface were proportional to the rates of change of $\alpha\Delta T$ and $\beta\Delta S$ respectively. A quantity of particular interest was the ratio of these fluxes, R_f, which at any time during a run was equal to the gradient of the graph of $\beta\Delta S$ against corresponding values of $\alpha\Delta T$.

Figure 2 shows graphs of $\beta\Delta S$ versus $\alpha\Delta T$ for all 10 runs. There is no significant departure of these graphs from a linear form, showing that the flux ratio was constant throughout each run. The values of the flux ratio determined from all the points of each run are shown in figure 3 plotted against the initial value $\beta\Delta S_0$ of $\beta\Delta S$, and it may be seen that there is a very slow increase of flux ratio with $\beta\Delta S_0$. The equation of the best-fit line through the points is

$$R_f = 0{\cdot}59 + 0{\cdot}18\beta\Delta S_0. \tag{3}$$

The r.m.s. deviation of the points from this line is 0·01, so that the variation with $\beta\Delta S_0$ is barely significant.

Two interesting points emerge from (3) when these results are compared with the corresponding heat–salt results of Turner. First, in that experiment R_f was independent of R_ρ only in the 'constant regime' when $R_\rho \gtrsim 2$. As R_ρ was reduced from 2 to its minimum value of unity R_f increased from 0·15 to 1·0 (the 'variable regime'). If this had happened in the salt–sugar experiment, the points in figure 2 would have fallen above the lines at the right-hand ends. There is no evidence for this behaviour, and it must be concluded either that the existence of a variable regime depended on the different conditions under which the heat–salt experiment was conducted (notably continuous heating from below), or that this behaviour would be noticed in the salt–sugar system only for $R_\rho \lesssim 1{\cdot}10$.

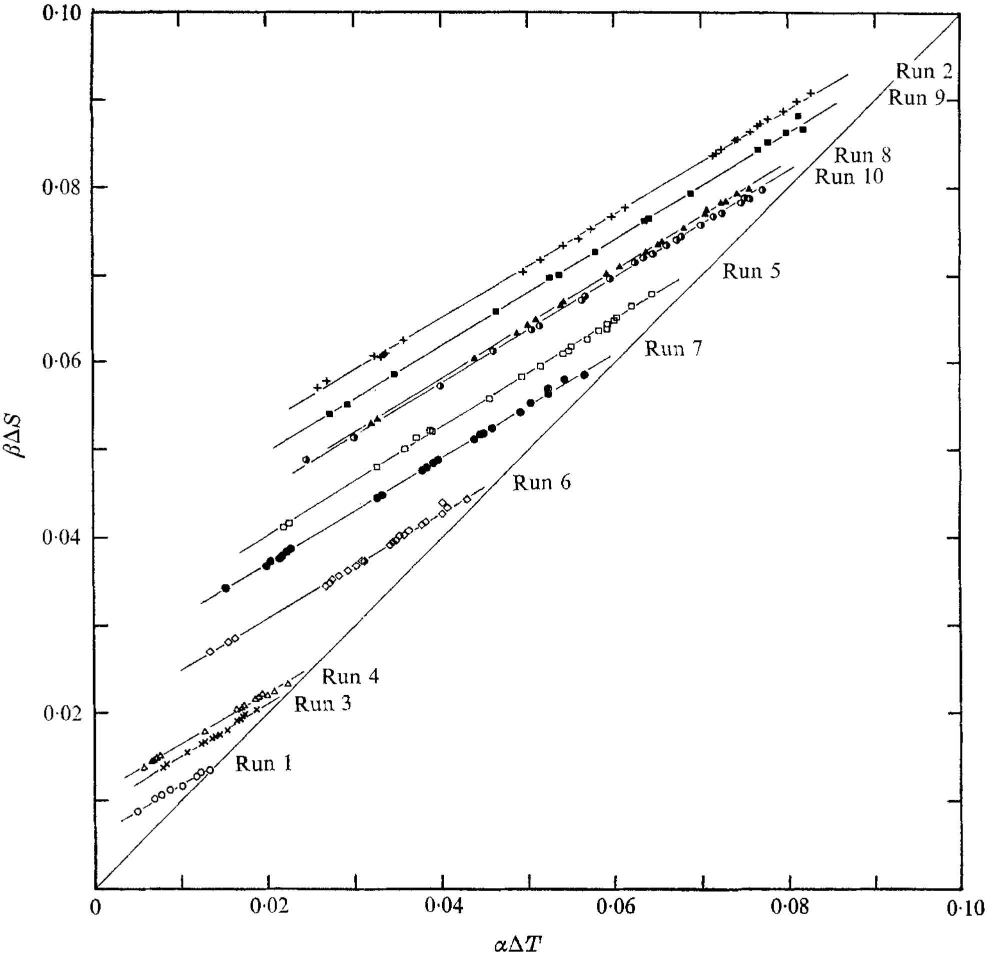

FIGURE 2. Graphs of $\beta\Delta S$ versus $\alpha\Delta T$ for all ten runs. The linear form of these graphs indicates that the ratio R_f of the S flux (sugar) to the T flux (salt) was constant throughout each run. The run number is indicated by each set of data, and the same symbols are used in figures 3–6.

Second, the value of R_f in the constant regime $R_\rho > 2$ for the heat–salt case was about 0·15, which corresponds within experimental uncertainty to $\tau^{\frac{1}{2}} = (\kappa_S/\kappa_T)^{\frac{1}{2}}$, where κ_S and κ_T are the molecular diffusivities for the two components. Veronis (1968) has outlined a justification for expecting this value of R_f when $R_\rho = 1$, but it is not understood why R_f should hold this value at higher values of R_ρ. It is, therefore, of interest to note that the salt–sugar system also obeys this relationship, within the uncertainty which attaches both to R_f and to $\tau^{\frac{1}{2}}$.

With regard to the uncertainty in the diffusivities, the values quoted earlier for salt and sugar are those taken from Weast (1968, p. F47), and they apply to very dilute solutions. The value of $\tau^{\frac{1}{2}}$ which they yield is 0·58, with an uncertainty probably about $\pm 5\%$, bearing in mind the differences which are usually found between the values reported by different sources for any one diffusivity. It seems

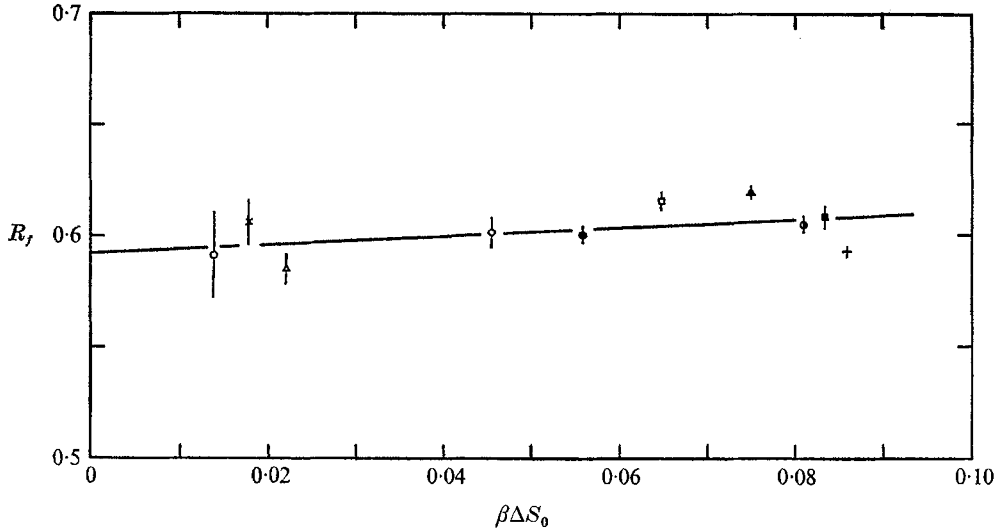

FIGURE 3. The dependence of the flux ratio R_f on the initial sugar concentration $\beta\Delta S_0$. (The error bars show the standard error in each determination.)

likely that the main effect of greater concentrations is to increase the viscosity of the solution, thereby reducing both diffusivities by the same fraction and leaving the ratio approximately constant. The reason for correlating R_f with $\beta\Delta S_0$ in figure 3 is that the viscosity of the liquid is controlled primarily by the sugar concentration. Thus the comparison between R_f and $\tau^{\frac{1}{2}}$ can best be made by extrapolating the line in figure 3 to give R_f when $\beta\Delta S_0$ is zero, approximating the conditions under which the diffusivities were measured. The standard error in R_f is 2 %, so that the two values of 0·59 and 0·58 respectively agree within the uncertainty of either. Figure 3 shows also that this value of R_f is only slightly altered by a wide variation of initial concentration $\beta\Delta S_0$, and hence of $\alpha\Delta T_0$, which was nearly equal to $\beta\Delta S_0$ in all runs; the small variation which does occur may probably be ascribed to a corresponding change in τ.

Just as the flux ratio depends on the diffusivity ratio, so also does the flux of either component individually. The flux of interest here is F_T, the flux of the faster-diffusing component which is driving the motion. In principle, F_T could be found by measuring the rate of change of $\alpha\Delta T$ with time. However, the experimental scatter in the values of $\alpha\Delta T$ prevented a reliable estimate of F_T based on differences between successive values and it was necessary to smooth the data before differentiating. It was not necessary to use an arbitrary smoothing function though; since F_T was controlled by $\alpha\Delta T$, a power-law relationship was plausible between $\alpha\Delta T$ and time t. Such a relationship is borne out by figure 4, which shows graphs for all runs of log $(\alpha\Delta T)$ versus $\log(t-t_0)$. The data all correlate well with straight lines, showing that we may write for any run

$$\alpha\Delta T \propto (t-t_0)^{-\gamma}. \tag{4}$$

In the absence of any reliable choice of the time origin t_0 on physical grounds, both t_0 and γ were chosen by a least-squares method to give the best fit. The fit

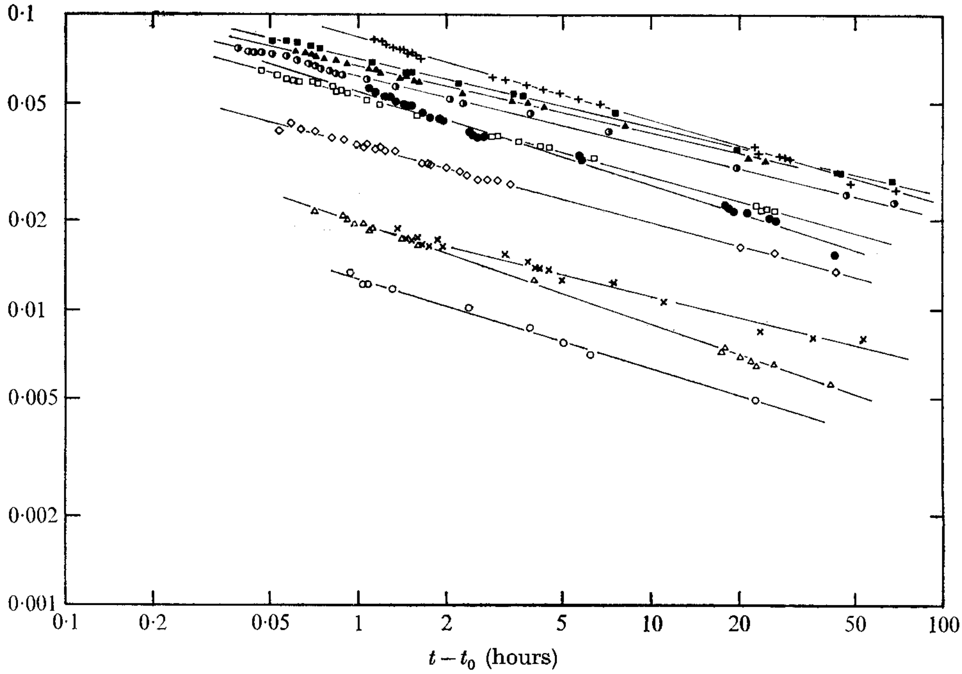

FIGURE 4. The decay of $\alpha\Delta T$ with time t. The lines correspond to the power law $\alpha\Delta T \propto (t-t_0)^{-\gamma}$, where t_0 and γ have been chosen by a least-squares method.

was quite satisfactory, the r.m.s. deviation for the lines in figure 4 lying between 1·4 % and 3·0 %, which is comparable with the uncertainty in measurement of $\alpha\Delta T$. However, by forcing the data to fit the function (4) some information has been lost at both ends of each run. Thus, although F_T was obtained as a smooth function of time by differentiating (4), this function must be treated with some caution at both the smallest and the largest values of $t-t_0$.

In order to focus attention on the dependence of F_T on R_ρ, its dependence on $(\alpha\Delta T)^{\frac{4}{3}}$ may be removed by writing (as did Turner)

$$Nu = 0{\cdot}085 Ra^{\frac{1}{3}} F_T^*,$$

where Nu is the Nusselt number corresponding to F_T, and Ra is the Rayleigh number for the T component, based on $\alpha\Delta T$ and the layer depth. Substituting for Nu and Ra gives

$$F_T = 0{\cdot}085 \rho g^{\frac{1}{3}} \nu^{-\frac{1}{3}} \kappa_T^{\frac{2}{3}} (\alpha\Delta T)^{\frac{4}{3}} F_T^*,$$

where ρ is the density, g the acceleration due to gravity and ν the kinematic viscosity. ρ, ν and κ_T all depend on the sugar and salt concentrations, and allowance was made for this in evaluating F_T^* from the measured flux F_T.

The resulting graph of F_T^* versus R_ρ for all runs is shown in figure 5. The accuracy of the data points is least where $\alpha\Delta T$ is small, particularly in runs 1, 3 and 4. Apart from these runs the error in F_T^* should not exceed 50 % when $R_\rho \doteqdot 1$, somewhat more when $R_\rho \gtrsim 2$, and somewhat less in between. The uncertainty in R_ρ varies from ± 3 % near $R_\rho = 1$ for large $\alpha\Delta T$ to about ± 10 % for large R_ρ and small $\alpha\Delta T$.

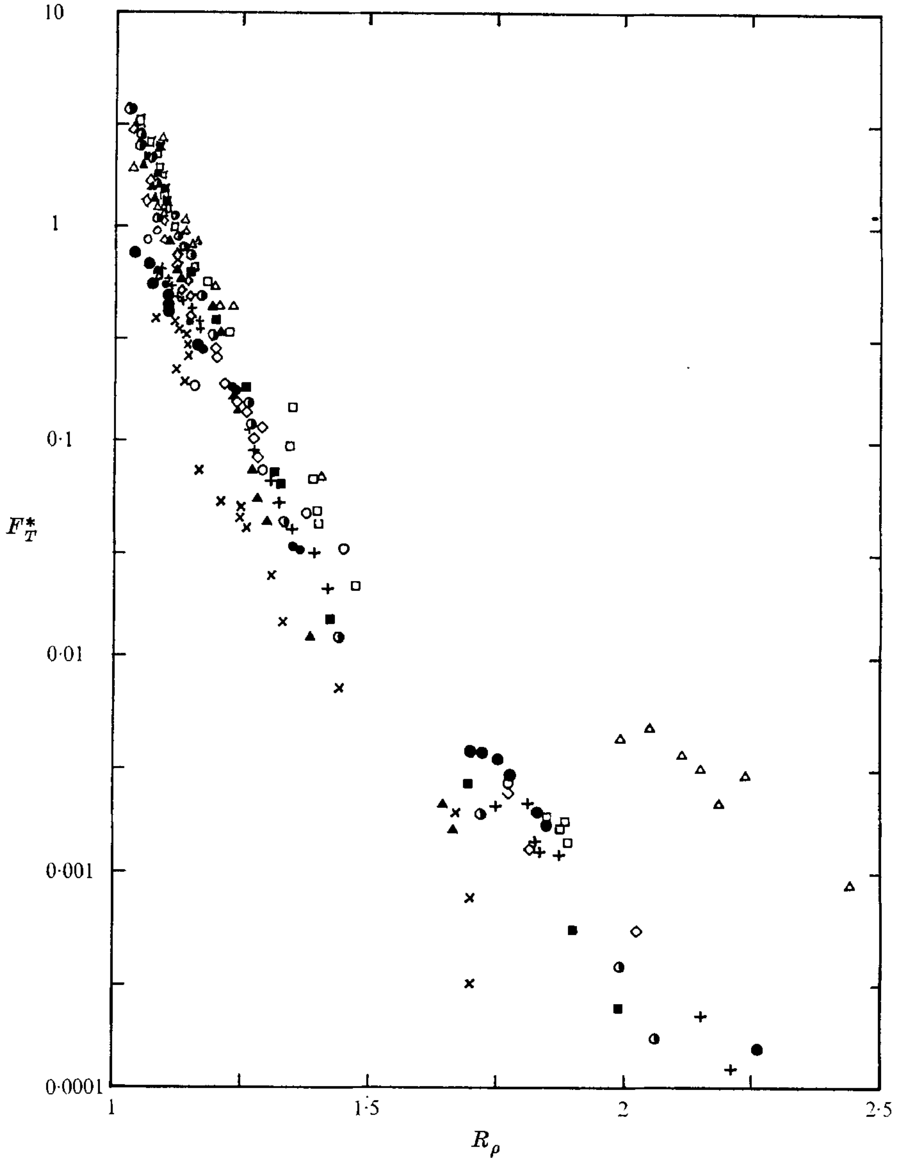

FIGURE 5. The normalized salt flux F_T^* versus the density anomaly ratio R_ρ for a salt–sugar diffusive interface.

Comparison of figure 5 with the corresponding graph of Turner (1965) shows the profound effect of varying τ. When this ratio is of order 10^{-2}, F_T^* decreases by two orders of magnitude as R_ρ increases from just above unity to 7; on the other hand, for a diffusivity ratio of 0·33, F_T^* decreases by four orders of magnitude when R_ρ increases only to 2.

Huppert (1971) has pointed out that the measurements of F_T^* for heat and salt correlate well with the function

$$F_T^* = \psi R_\rho^\phi, \tag{5}$$

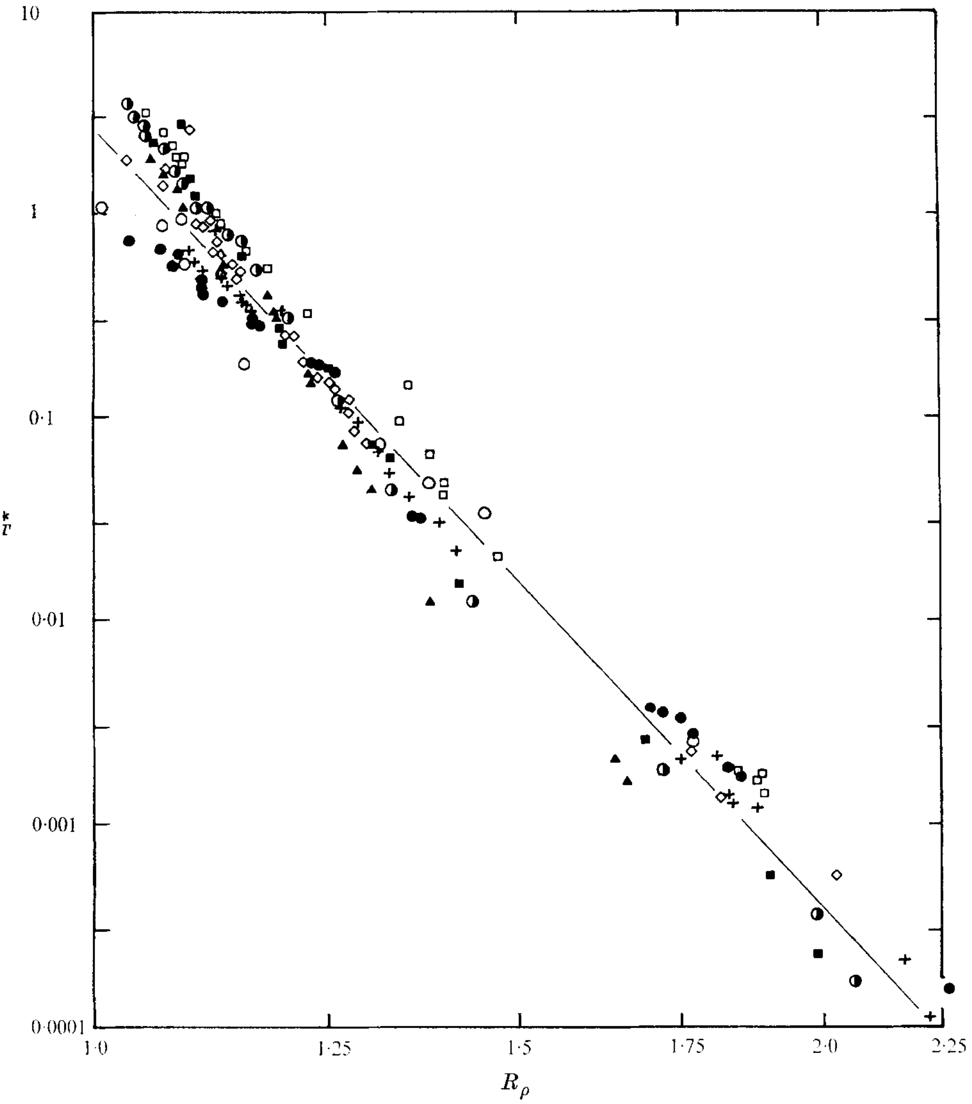

FIGURE 6. As figure 5, but redrawn as a log–log plot with the least reliable points omitted. The line corresponds to the function $F_T^* = \psi R_p^{\phi}$, where ψ (= 2·6) and ϕ (= $-12·6$) have been chosen by least-squares to fit the range $1·1 \leqslant R \leqslant 2$.

where $\phi = -2$. This relationship clearly fails for salt and sugar, but it is worth seeking other values of the parameters which will allow (5) to be retained.

Figure 6 shows the data of figure 5 replotted on a log–log graph, with runs 3 and 4 omitted on account of their disagreement in figure 5 and their low reliability. The line drawn in figure 6 has been fitted by a least-squares method to the 118 data points which lie in the range $1·1 \leqslant R \leqslant 2$, the extremes of R_p being excluded because of the unreliability of the corresponding flux measurements. All points in figure 6 have been given equal weight, and the resulting values are $\psi = 2·59$ and $\phi = -12·6$. Within the accuracy of the data the fit is good and no higher-order function is justified.

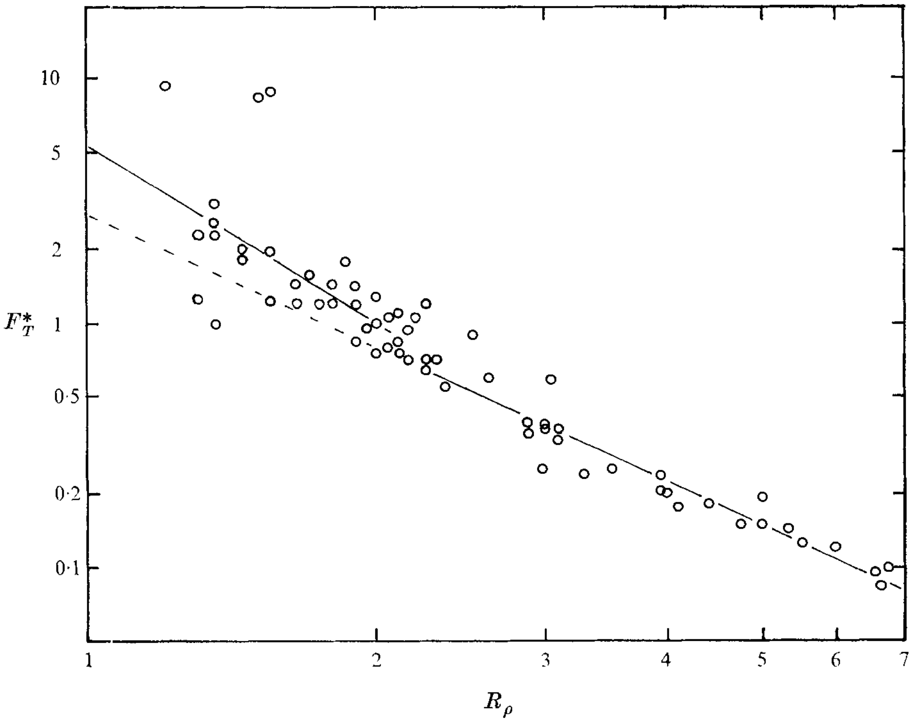

FIGURE 7. Results published by Turner (1965) for the normalized heat flux F_T^* versus R_ρ, for a diffusive interface in which heat and salt were the diffusing components. In this case the data separate into a constant regime ($R_f = 0{\cdot}15$ for $R_\rho \gtrsim 2$) and a variable regime ($R_f > 0{\cdot}15$, $1 < R_\rho \lesssim 2$). The two lines drawn through the data are least-squares fits for the two regimes separately. In the constant regime, $F_T^* = \psi R_\rho^\phi$, where $\psi = 2{\cdot}67$ and $\phi = -1{\cdot}79$ (cf. figure 6).

To provide a proper comparison with the heat–salt measurements, figure 7 shows Turner's results plotted in the same way as the salt–sugar results in figure 6. While ϕ was determined from figure 6 excluding only that data which was considered unreliable, in the case of the heat–salt data there is physical justification for dividing the data into two sets which correspond respectively to the variable regime $R_\rho \lesssim 2$ and the constant regime $R_\rho \gtrsim 2$. Two lines corresponding to each of these sets individually are drawn through the data in figure 7. In order to prevent contamination of the data corresponding to the constant regime by points belonging to the variable regime, the line for the former was calculated by a least-squares method using points with $R_\rho \geqslant 2{\cdot}3$, and this line has the parameters $\psi = 2{\cdot}67$, $\phi = -1{\cdot}79$. The points from the variable regime $R_\rho \leqslant 2$ are so scattered that the line through that set of data has no physical significance, but the line for the constant regime may be compared directly with that obtained in the salt–sugar experiment, figure 6.

Thus all the experimental data currently available on the constant regime of the diffusive interface can be accommodated in the simple relationship (5), wherein ϕ is determined in some way as yet unknown by the diffusivity ratio τ, while ψ is, within the accuracy of existing measurements, constant. The two values for ψ, namely 2·59 and 2·67, are not significantly different from the value 2·3 which Huppert (1971) has suggested one should expect. The reason for this value is that the fluxes have been non-dimensionalized by reference to the flux through a system with rigid horizontal boundaries, whereas here we have boundaries which are approximately stress-free.

4. Profile measurements

Flux measurements such as those described in the preceding sections are of considerable phenomenological interest, but do not by themselves help very much to determine how the diffusive interface works. In an attempt to gain some insight into the structure of the interface, as some guide to theoretical studies, vertical profiles of horizontal-mean concentrations were measured through a salt–sugar diffusive interface.

The experiment was carried out in a Perspex tank 15 cm long and 2·5 cm wide. Measurements were made optically, with the light beam parallel to the short sides of the tank. This dimension had to be kept small in order to limit the deflexion of the beam in the interface. The apparatus is illustrated in figure 8, and used the polarized beam emitted horizontally from a 1 mW helium–neon laser.

The tank could be moved vertically past the beam, which was 0·3 cm in diameter. A horizontal slit, which could be accurately positioned vertically, was placed in the focal plane of a lens situated beyond the tank. When this slit was set to allow passage of the beam, its position indicated the angle through which the beam was deflected in passing through the tank, and hence the mean vertical gradient of refractive index along the light path. After passing through the slit, the light was incident on a nicol prism analyser (Bellingham & Stanley Model A/P1 analyser head) with which the rotation of the plane of polarization was determined. The telescope in the analyser was focused on a scale with 0·2 cm graduations, which stood against the side of the tank on which the light was incident. The level in the tank at which the measurements of sugar concentration and refractive index gradient were taken could be determined by viewing this scale directly, at the same time as the analyser and slit were adjusted.

The focal length of the lens was 15 cm, and the slit could be set within $\pm 0{\cdot}0025$ cm. The uncertainty in angular deflexion of the beam was therefore $\pm 3 \times 10^{-4}$ radians, and the corresponding uncertainty in refractive index gradient was $\pm 1 \times 10^{-4}\,\mathrm{cm}^{-1}$ provided that the deflexion was not too large. However, in the early stages of the experiment the large deflexions which were produced within a very thin region of the interface introduced systematic errors which were much larger than this; useful readings could only be taken when the vertical displacement of the beam within the tank was less than the interface thickness.

The accuracy with which the analyser could be operated depended primarily on the light intensity. The tank was made of normalized Perspex and was annealed after construction, to reduce the depolarizing effects of internal stresses. While these precautions were not entirely successful, it was only very near the top and bottom of the tank that the uncertainty in setting the analyser exceeded the uncertainty in reading it, $\pm 0{\cdot}005^\circ$, provided that the light was sufficiently

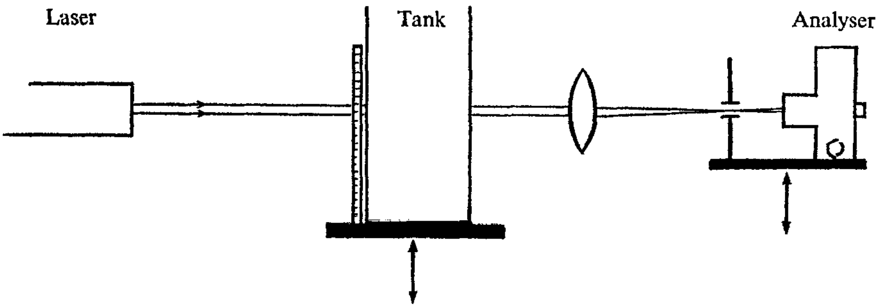

FIGURE 8. The optical system used for measuring vertical profiles of horizontal-average sugar and salt concentrations.

bright. In the early stages of the experiment, however, some difficulty was experienced in taking readings near the interface. Here only a small fraction of the available light in the beam actually passed through the slit, owing to the rapid variation of deflexion of the beam with vertical position in the tank, and the image was, therefore, rather dim. Under these conditions the eye could not define so well the minimum in brightness as the nicol was rotated. The uncertainty then exceeded ± 0·005°, but was always less than ± 0·05°, corresponding to an uncertainty of ± 0·002 in βS.

Each profile measured during the experiment was made up of about 30 measurements, usually at vertical intervals of 0·4 cm. Near the interface readings were taken closer together than this, and it was necessary to have a more accurate measure of the vertical level z to which they should be ascribed than was provided by the scale on the tank. A horizontal needle was mounted in front of the scale on a micrometer mechanism, which was fastened to the platform on which the tank rested. The needle projected half-way across the light beam and could be seen through the analyser telescope. Thus in any case where it was important to know z accurately at a level other than at one of the scale graduations, it could be found easily within ± 0·005 cm.

The experiment began with a salt solution of density 1·077 g cm^{-3} floated onto a sugar solution of density 1·095 g cm^{-3}. (However, the effective initial values of $\alpha\Delta T$ and $\beta\Delta S$ were less than 0·077 and 0·095 respectively by indeterminate amounts, owing to the disturbance of the interface while the salt solution was being poured.) Initially the gradients of both components in the interface were so large that, for the reasons indicated above, no useful measurements could be made. The first valid measurement was the profile determined 8·2 h after the experiment began. A total of six such measurements was made, the last being at 97 h.

The angular deflexion of a horizontal light beam, which enters a tank of liquid in which the refractive index varies only in the vertical direction, is proportional to the vertical gradient of refractive index to first order (the second-order corrections have been considered by Shirtcliffe (1969*b*)). Each set of measured deflexions was, therefore, integrated numerically to give a set of values of refractive index in the tank. The end points of the integration were set by the requirement

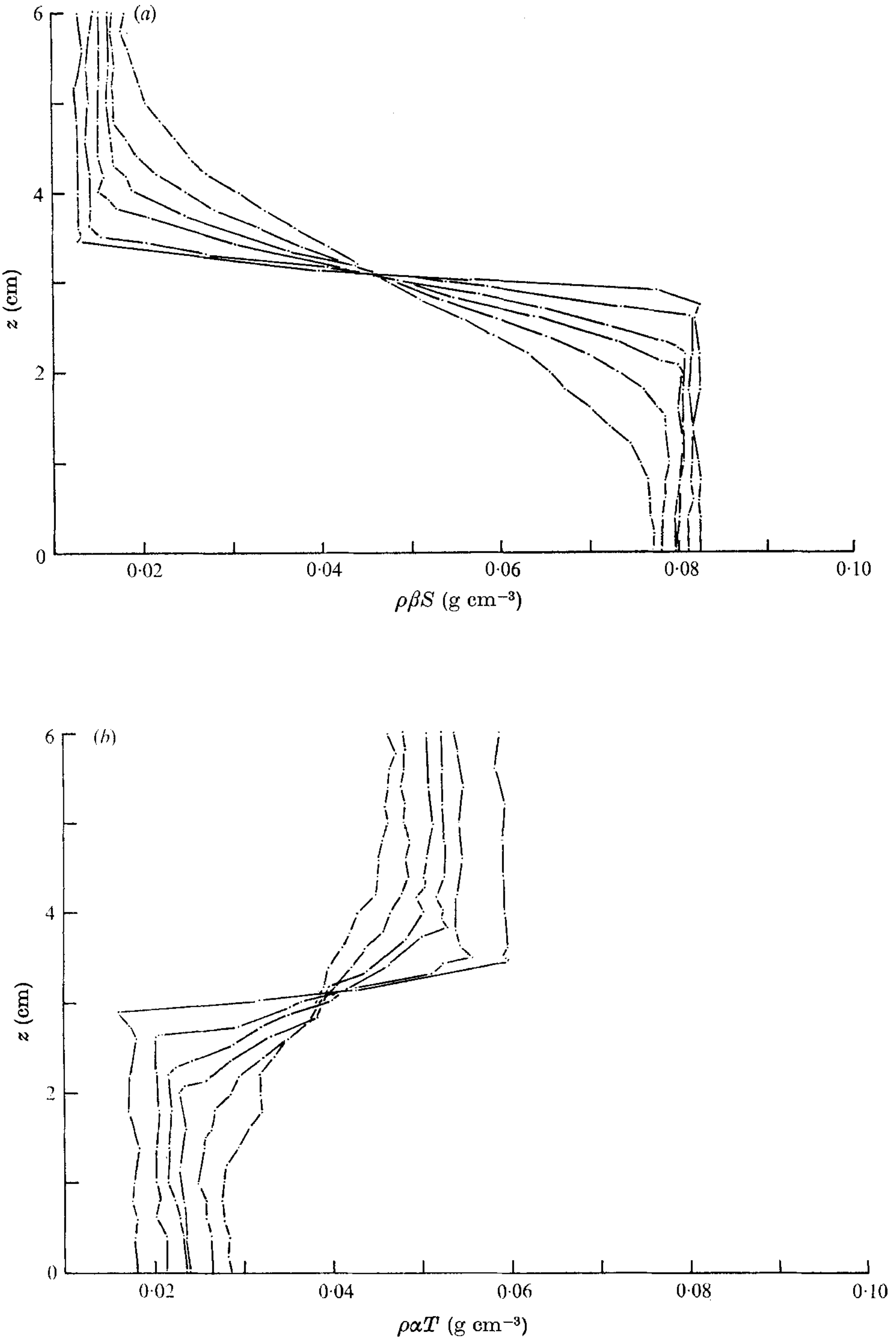

FIGURE 9. For legend see facing page.

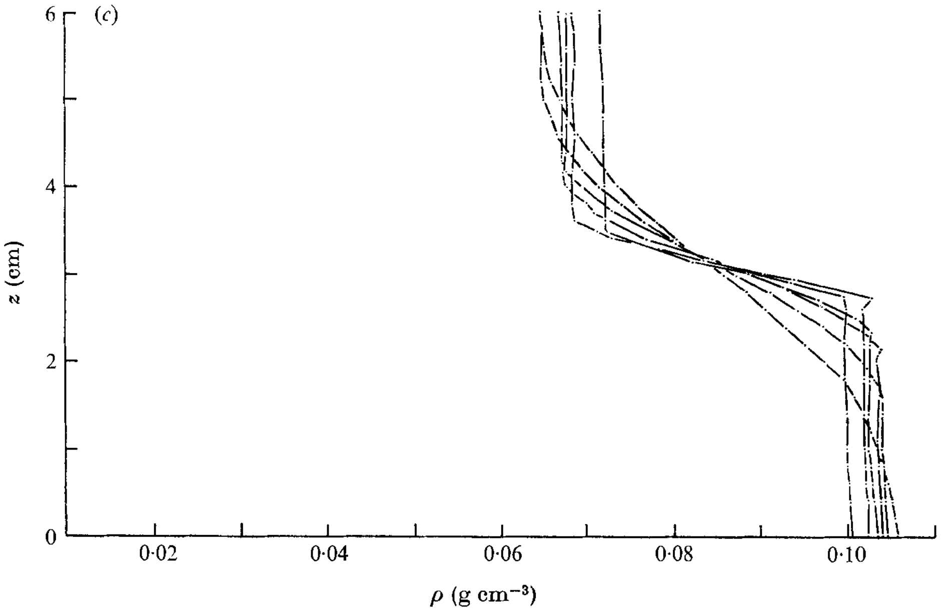

FIGURE 9. The measured profiles of (*a*) sugar concentration, (*b*) salt concentration and (*c*) density. The times at which the measurements were made are given in table 1.

Time (h)	$\alpha\Delta T$	$\beta\Delta S$	$\rho\alpha dT/dz$ (g cm^{-4})	F_{TC} (10^{-7} g cm^{-2} s^{-1})	F_T (10^{-7} g cm^{-2} s^{-1})	$\rho\beta dS/dz$ (g cm^{-4})	F_{SC} (10^{-7} g cm^{-2} s^{-1})	F_{SC}/F_{TC}
8·2	0·041	0·07	0·095	7·5	6·8	0·132	3·5	0·5
12	0·034	0·067	0·043	3·4	4·1	0·075	2·0	0·6
24	0·03	0·065	0·022	1·82	1·67	0·052	1·43	0·8
32·3	0·027	0·063	0·015	1·24	1·13	0·037	1·02	0·8
53·5	0·022	0·061	0·010	0·83	0·58	0·027	0·75	0·9
96·5	0·018	0·058	0·006	0·46	0·27	0·018	0·46	1·0

TABLE 1. A comparison of the observed flux F_T with the fluxes F_{TC} and F_{SC} which would have been produced by the vertical gradients of the S and T components observed within the interface. The observed flux F_S was given by $F_S/F_T = 0{\cdot}6$ throughout.

that the mean refractive index had to be that for a solution of the known total quantities of sugar and salt in the water.

The fact that measurements had to be taken throughout the vertical extent of both layers in the tank limited the maximum layer depth which could be accommodated, in part so that the tank traverse mechanism could be kept simple, and in part to ensure that the measurement of any one profile could be completed quickly in relation to the rate of change of concentrations. Both layers were, therefore, made 3 cm deep.

Such a small layer depth had the disadvantage that the interface thickness increased rather rapidly to a value which was not negligible by comparison, producing some problems of interpretation. The profiles of sugar and salt concentration, and of density, are drawn in figures 9(*a*), (*b*) and (*c*). It may be seen that within the interface these profiles were approximately linear at first, but became less so as the interface became thicker.

Fluxes through the interface were determined as they were in the flux experiment, by fitting the function (4) to the data. Measurements of $\alpha\Delta T$ were less precise in this experiment, since they relied on the integration of a series of gradient measurements whose errors were cumulative. The times at which profiles were measured, and the corresponding measurements of the flux F_T, are shown in table 1.

This experiment allowed measurements of the gradients of density attributable to each component, that is $\rho\alpha\, dT/dz$ and $\rho\beta\, dS/dz$, within the interface. These quantities are also included in table 1, together with the calculated fluxes F_{TC} and F_{SC} which these gradients would induce owing to molecular diffusion alone. In the first two profiles, these quantities are each averaged over measurements at three levels within the interface; in the remaining four profiles, the values correspond to a measurement near the mid-plane of the interface.

The final column in table 1 shows the ratio F_{SC}/F_{TC}. For comparison, it should be noted that the plot of ΔT versus ΔS for this experiment indicated that the flux ratio $R_f = F_S/F_T$ was substantially constant and consistent with the value 0·6 measured in the flux experiment. Initially, F_{SC}/F_{TC} was not significantly different from R_f. However, while R_f remained constant, F_{SC}/F_{TC} increased steadily toward unity. This was a consequence of the non-steady nature of the experiment, and the increasing thickness of the interface; the time scale for diffusion became so long that F_{TC} and F_{SC} fell off less rapidly than F_T and F_S. The effect was greater for F_{SC}, since the diffusivity is smaller for the S component, so that F_{SC}/F_{TC} increased with time. If the experiment had been conducted in a more nearly steady state, using deeper layers, there is little doubt that equality would have been maintained between F_{TC} and F_T, between F_{SC} and F_S and between F_{SC}/F_{TC} and R_f.

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